

GCE

Mathematics B (MEI)

H640/02: Pure Mathematics and Statistics

Advanced GCE

Mark Scheme for Autumn 2021

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This mark scheme is published as an aid to teachers and students, to indicate the requirements of the examination. It shows the basis on which marks were awarded by examiners. It does not indicate the details of the discussions which took place at an examiners' meeting before marking commenced.

All examiners are instructed that alternative correct answers and unexpected approaches in candidates' scripts must be given marks that fairly reflect the relevant knowledge and skills demonstrated.

Mark schemes should be read in conjunction with the published question papers and the report on the examination.

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1. Annotations and abbreviations

Annotation in scoris	Meaning
✓ and ✖	
BOD	Benefit of doubt
FT	Follow through
ISW	Ignore subsequent working
M0, M1	Method mark awarded 0, 1
A0, A1	Accuracy mark awarded 0, 1
B0, B1	Independent mark awarded 0, 1
E	Explanation mark 1
SC	Special case
^	Omission sign
MR	Misread
BP	Blank page
Highlighting	
Other abbreviations in mark scheme	Meaning
E1	Mark for explaining a result or establishing a given result
dep*	Mark dependent on a previous mark, indicated by *. The * may be omitted if only previous M mark.
cao	Correct answer only
oe	Or equivalent
rot	Rounded or truncated
soi	Seen or implied
www	Without wrong working
AG	Answer given
awrt	Anything which rounds to
BC	By Calculator
DR	This indicates that the instruction In this question you must show detailed reasoning appears in the question.

2. Subject-specific Marking Instructions for AS/A Level Mathematics B (MEI)

- a a Annotations must be used during your marking. For a response awarded zero (or full) marks a single appropriate annotation (cross, tick, M0 or ^) is sufficient, but not required.

For responses that are not awarded either 0 or full marks, you must make it clear how you have arrived at the mark you have awarded and all responses must have enough annotation for a reviewer to decide if the mark awarded is correct without having to mark it independently.

It is vital that you annotate standardisation scripts fully to show how the marks have been awarded.

Award NR (No Response)

- if there is nothing written at all in the answer space and no attempt elsewhere in the script
- OR if there is a comment which does not in any way relate to the question (e.g. 'can't do', 'don't know')
- OR if there is a mark (e.g. a dash, a question mark, a picture) which isn't an attempt at the question.

Note: Award 0 marks only for an attempt that earns no credit (including copying out the question).

If a candidate uses the answer space for one question to answer another, for example using the space for 8(b) to answer 8(a), then give benefit of doubt unless it is ambiguous for which part it is intended.

- b An element of professional judgement is required in the marking of any written paper. Remember that the mark scheme is designed to assist in marking incorrect solutions. Correct solutions leading to correct answers are awarded full marks but work must not always be judged on the answer alone, and answers that are given in the question, especially, must be validly obtained; key steps in the working must always be looked at and anything unfamiliar must be investigated thoroughly. Correct but unfamiliar or unexpected methods are often signalled by a correct result following an apparently incorrect method. Such work must be carefully assessed. When a candidate adopts a method which does not correspond to the mark scheme, escalate the question to your Team Leader who will decide on a course of action with the Principal Examiner. If you are in any doubt whatsoever you should contact your Team Leader.

- c The following types of marks are available.

M

A suitable method has been selected and *applied* in a manner which shows that the method is essentially understood. Method marks are not usually lost for numerical errors, algebraic slips or errors in units. However, it is not usually sufficient for a candidate just to indicate an intention of using some method or just to quote a formula; the formula or idea must be applied to the specific problem in hand, e.g. by substituting the relevant quantities into the formula. In some cases the nature of the errors allowed for the award of an M mark may be specified.

A method mark may usually be implied by a correct answer unless the question includes the DR statement, the command words “Determine” or “Show that”, or some other indication that the method must be given explicitly.

A

Accuracy mark, awarded for a correct answer or intermediate step correctly obtained. Accuracy marks cannot be given unless the associated Method mark is earned (or implied). Therefore M0 A1 cannot ever be awarded.

B

Mark for a correct result or statement independent of Method marks.

E

A given result is to be established or a result has to be explained. This usually requires more working or explanation than the establishment of an unknown result.

Unless otherwise indicated, marks once gained cannot subsequently be lost, e.g. wrong working following a correct form of answer is ignored. Sometimes this is reinforced in the mark scheme by the abbreviation isw. However, this would not apply to a case where a candidate passes through the correct answer as part of a wrong argument.

- d When a part of a question has two or more ‘method’ steps, the M marks are in principle independent unless the scheme specifically says otherwise; and similarly where there are several B marks allocated. (The notation ‘dep*’ is used to indicate that a particular mark is dependent on an earlier, asterisked, mark in the scheme.) Of course, in practice it may happen that when a candidate has once gone wrong in a part of a question, the work from there on is worthless so that no more marks can sensibly be given. On the other hand, when two or more steps are successfully run together by the candidate, the earlier marks are implied and full credit must be given.
- e The abbreviation FT implies that the A or B mark indicated is allowed for work correctly following on from previously incorrect results. Otherwise, A and B marks are given for correct work only – differences in notation are of course permitted. A (accuracy) marks are not given for answers obtained from incorrect working. When A or B marks are awarded for work at an intermediate stage of a solution, there may be various alternatives that are equally acceptable. In such cases, what is acceptable will be detailed in the mark scheme. If this is not the case, please escalate the question to your Team Leader who will decide on a course of action with the Principal Examiner.
- Sometimes the answer to one part of a question is used in a later part of the same question. In this case, A marks will often be ‘follow through’. In such cases you must ensure that you refer back to the answer of the previous part question even if this is not shown within the image zone. You may find it easier to mark follow through questions candidate-by-candidate rather than question-by-question.

- f Unless units are specifically requested, there is no penalty for wrong or missing units as long as the answer is numerically correct and expressed either in SI or in the units of the question. (e.g. lengths will be assumed to be in metres unless in a particular question all the lengths are in km, when this would be assumed to be the unspecified unit.)

We are usually quite flexible about the accuracy to which the final answer is expressed; over-specification is usually only penalised where the scheme explicitly says so.

- When a value is **given** in the paper only accept an answer correct to at least as many significant figures as the given value.
- When a value is **not given** in the paper accept any answer that agrees with the correct value to **2 s.f.** unless a different level of accuracy has been asked for in the question, or the mark scheme specifies an acceptable range.

NB for Specification A the rubric specifies 3 s.f. as standard, so this statement reads “3 s.f”

Follow through should be used so that only one mark in any question is lost for each distinct accuracy error.

Candidates using a value of 9.80, 9.81 or 10 for g should usually be penalised for any final accuracy marks which do not agree to the value found with 9.8 which is given in the rubric.

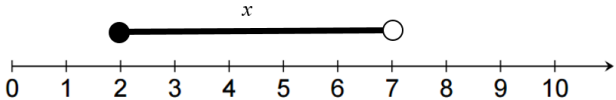
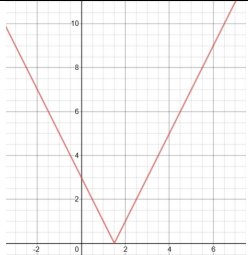
- g Rules for replaced work and multiple attempts:

- If one attempt is clearly indicated as the one to mark, or only one is left uncrossed out, then mark that attempt and ignore the others.
- If more than one attempt is left not crossed out, then mark the last attempt unless it only repeats part of the first attempt or is substantially less complete.
- if a candidate crosses out all of their attempts, the assessor should attempt to mark the crossed out answer(s) as above and award marks appropriately.

- h For a genuine misreading (of numbers or symbols) which is such that the object and the difficulty of the question remain unaltered, mark according to the scheme but following through from the candidate’s data. A penalty is then applied; 1 mark is generally appropriate, though this may differ for some units. This is achieved by withholding one A or B mark in the question. Marks designated as cao may be awarded as long as there are no other errors. If a candidate corrects the misread in a later part, do not continue to follow through. E marks are lost unless, by chance, the given results are established by equivalent working. Note that a miscopy of the candidate’s own working is not a misread but an accuracy error.

- i If a calculator is used, some answers may be obtained with little or no working visible. Allow full marks for correct answers provided that there is nothing in the wording of the question specifying that analytical methods are required such as the bold “In this question you must show detailed reasoning”, or the command words “Show” and “Determine. Where an answer is wrong but there is some evidence of method, allow appropriate method marks. Wrong answers with no supporting method score zero. If in doubt, consult your Team Leader.

- j If in any case the scheme operates with considerable unfairness consult your Team Leader.

Question		Answer	Marks	AOs	Guidance
1		$(y =) 4(kx)^2 + 8(kx) + 1$ soi	M1	1.1a	$k = \frac{1}{2}$; condone $k = 2$; condone omission of brackets
		$y = x^2 + 4x + 1$ isw	A1	1.1	must see “y =” either here or in first line
			[2]		
2 (a)		$65 \times \frac{\pi}{180}$ soi	M1	1.1a	may be implied by 1.13(4464...) or $0.36... \times \pi$
		$\frac{13\pi}{36}$	A1	1.1	
			[2]		
2 (b)		12.1	B1	1.1	ignore units
			[1]		
3			M1	1.2	line from 2 to 7 only (do not need number line)
			A1	1.1	one end correctly shaded
			A1	2.5	both ends correctly shaded
			[3]		
4			M1	1.1	$y = 2x - 3$ correct in first quadrant; may be implied by reflection or by A1
			M1	1.1	reflection of their straight line to form V shape for positive y only
			A1	1.1	symmetrical V shape through (0,3) with vertex at (1.5, 0); points may be identified from numbered scale or written separately
			[3]		
5 (a)		0.14, 0.139 or awrt 0.1385 isw BC	B1	1.1	
			[1]		

Question		Answer	Marks	AOs	Guidance
5	(b)	$P(X \leq 7) = 0.0248(2\dots)$ soi or 0.9751 to 0.9752 seen	M1	1.1	if M0 , allow SC1 for awrt 0.0575... or 0.943
		0.975 cao	A1	1.1	
			[2]		
6	(a)	$\left(\frac{2 \times 5 + 3 \times (-1)}{2 \times 3 + 3 \times 6}\right)$ soi	M1	3.1a	may be implied if one component fully correct
		$\sqrt{7^2 + 24^2}$	M1	1.1	dependent on award of first M1
		25	A1	1.1	
			[3]		
6	(b)	$\tan^{-1}\left(\frac{24}{7}\right)$ oe	M1	1.1a	or $\cos^{-1}\left(\frac{7}{25}\right)$ or $\sin^{-1}\left(\frac{24}{25}\right)$ FT their 7, 24 or 25
		74° or 73.7° or awrt 73.74° or 1.3 or 1.29 or awrt 1.287	A1	1.1	
			[2]		
7	(a)	$(x - 7)^2 + (y + 3)^2 = 5^2 \cos^2 \theta + 5^2 \sin^2 \theta$ oe	M1	3.1a	allow sign error
		use of $\cos^2 \theta + \sin^2 \theta = 1$ to eliminate θ	M1	1.1	
		<i>Alternative</i> centre is (7, -3) and substituted in correct form of equation	M1		
		radius is 5 and substituted in correct form of equation	M1		
		$(x - 7)^2 + (y + 3)^2 = 5^2$ oe isw	A1	1.1	if M0M0 allow SC1 for $y = 5 \sin \left\{ \cos^{-1} \left(\frac{x - 7}{5} \right) \right\} - 3$ $x = 5 \cos \left\{ \sin^{-1} \left(\frac{y + 3}{5} \right) \right\} + 7$
			[3]		
7	(b)	(7, -3)	B1	2.2a	FT their $(x - 7)^2 + (y + 3)^2 = 25$
			[1]		

Question		Answer	Marks	AOs	Guidance
8		$b^2 = \frac{11.492}{6.8}$ oe	M1	3.1a	
		$b = 1.3$	A1	1.1	
		$42b + a = 57.2$	M1	1.1	b may be incorrect numerical value
		$a = 2.6$	A1	3.2a	
			[4]		
9	(a)	$\frac{5}{9} \times \frac{4}{8} \times \frac{3}{7} \times \frac{4}{6}$ oe $\times 4$	M1 M1	1.1 1.1	M0 if binomial distribution or sampling with replacement used dependent on award of first M1
		<i>Alternative</i> ${}_9C_4 [= 126]$ soi	M1	1.1	
		${}_5C_3 \times {}_4C_1 [= 10 \times 4]$ soi	M1	1.1	
		$\frac{40}{126}$ or $\frac{20}{63}$ or 0.317460...isw or rounded to 2 sf or better	A1	1.1	
			[3]		

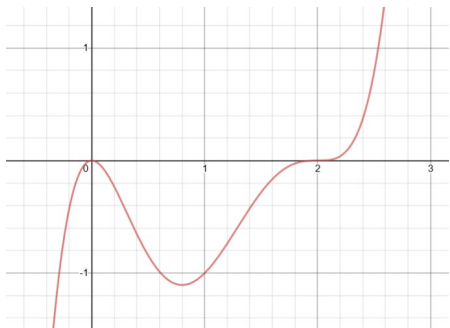
Question		Answer	Marks	AOs	Guidance
9	(b)	$\frac{4}{9} \times \frac{3}{8} \times \frac{2}{7} \times \frac{5}{6} \times 4$ oe or $\frac{4}{9} \times \frac{3}{8} \times \frac{2}{7} \times \frac{1}{6}$ oe	M1	3.1b	condone omission of $\times 4$ in first term; M0 if binomial distribution or sampling with replacement used
			M1	1.1	for addition of their terms; dependent on award of first M1
		<i>Alternative</i> ${}_4C_4 + {}_4C_3 \times {}_5C_1$	M1 M1		for ${}_4C_3 \times {}_5C_1$ for addition of their terms; dependent on award of first M1 NB $1 + 4 \times 5 = 21$
		<i>Alternative</i> $\frac{5}{9} \times \frac{4}{8} \times \frac{3}{7} \times \frac{2}{6} + 4 \times \frac{4}{9} \times \frac{5}{8} \times \frac{4}{7} \times \frac{3}{6} + 6 \times \frac{4}{9} \times \frac{5}{8} \times \frac{4}{7} \times \frac{3}{6}$ oe 1 – their $\frac{2520}{3024}$	M1 M1		for two of these terms for 1 – the sum of their 3 terms NB $\frac{2520}{3024} = \frac{5}{6}$
		$\frac{1}{6}$ or 0.166666...to 2 sf or better	A1	1.1	
			[3]		
9	(c)	(3 BF, 1M) + (2BF, 1NBF, 1BM) attempted	M1	3.1b	M0 if binomial distribution or sampling with replacement used
			A1	1.1	
		$\frac{3}{9} \times \frac{2}{8} \times \frac{1}{7} \times \frac{4}{6} \times 4 + \frac{3}{9} \times \frac{2}{8} \times \frac{2}{7} \times \frac{1}{6} \times 12 \left[= \frac{5}{63} \right]$ oe	A1	1.1	
		<i>Alternative</i> ${}_3C_3 \times {}_4C_1 + {}_3C_2 \times {}_2C_1 \times {}_1C_1$ 10	M1 A1		for either term
		$\frac{10}{21}$ or 0.476190476...to 2 sf or better	A1	1.1	
			[3]		

Question		Answer	Marks	AOs	Guidance	
9	(d)	not independent since $\frac{10}{21} \neq \frac{20}{63}$ oe	B1	2.4	FT their probabilities	no FT if binomial distribution or sampling with replacement used
		<i>Alternatively</i> not independent since $\frac{20}{63} \times \frac{1}{6} \neq \frac{5}{63}$			FT their calculated probabilities from first three parts	
			[1]			
10	(a)	to account for seasonal variation in birds oe or to account for different weather conditions which affects what birds are seen oe	B1	2.2b		
			[1]			
10	(b)	positive skew cao	B1	1.2		
			[1]			
10	(c)	eg in lower quartile (or lower tail) eg much lower than median (or middle) oe eg frequency of 3 and others have frequency of 2 eg distribution is skewed (so median more appropriate)	B1	2.4	NB mode is 9	
			[1]			
10	(d)	median is 19.5	B1	1.1		
		LQ is between 8 th and 9 th value or UQ is between 24 th and 25 th value soi	M1	1.1	allow for sight of 11 or 30	
		IQR = 30 – 11 = 19	A1	1.1		
			[3]			
10	(e)	2015: $32 \leq \text{median} \leq 34$, much bigger than (their 19.5) in 2019 which supports Ben's first statement oe 2015: $31 \leq \text{IQR} \leq 34$ was (much) bigger than (their 19) in 2019 which does not support Ben's second statement oe	B1 B1	2.2b 2.4	if B0B0 allow SC1 for true – median smaller in 2019 oe and false – IQR smaller in 2019 oe	
			[2]			

Question		Answer	Marks	AOs	Guidance	
10	(f)	eg other distributions are symmetrical (so could be eg Normal) oe eg can't tell how values distributed in each quartile oe	B1	2.4		
			[1]			
11	(a)	$H_0 : \mu = 161.6$ $H_1 : \mu > 161.6$	B1	1.1	both hypotheses; allow other parameters (but not X or \bar{X}) if defined as mean	
		μ is the population mean height of adult females (in 2020)	B1	2.5		
		1-tailed test because it is suspected that the mean height has increased oe	B1	2.4		
			[3]			
11	(b)	use of $\bar{X} \square N\left(161.6, \frac{1.96}{200}\right)$ soi	M1	3.3	may see $1.645 = \frac{\bar{X} - 161.6}{\sqrt{\frac{1.96}{200}}}$	$\bar{X} = 161.76 \dots$ implies M1 only
		awrt $\bar{X} > 161.8$ BC	A1	1.1	allow if inequality not strict NB 161.76283...	allow eg sample mean > 161.8
			[2]			

Question		Answer	Marks	AOs	Guidance	
11	(c)	the 1 st statement is correct because $\bar{X} > 161.8$ or $\bar{X} > 161.8$ so the sample mean is in the critical region	M1	2.3	FT their 161.8 (must be greater than 161.6) or allow if <i>their</i> $P(\bar{X} > 161.9) [= 0.00122] < 0.05$ or <i>their</i> $z > 1.645$ is considered	critical region must be an upper tail; their probability must be less than 0.5
		the 2 nd statement is incorrect because the sample mean is in the critical region oe or because the result is significant oe or ‘...so/hence the null hypothesis is rejected.’ following from the first statement	A1	2.4	FT their calculated critical region or their calculated probability,	
		the 3 rd statement is incorrect; because it is only possible to infer, not prove, using a hypothesis test oe	B1	2.2b	ignore comments about rejecting the null hypothesis oe	
			[3]			
12	(a)	#N/A is used to stop the software reading the entry as zero	B1	2.4	allow so that the cell is ignored oe or the software interprets #N/A as no data oe	advantage
			[1]			
12	(b)	(30.6, 128) and (1.3,88) ringed	B1	1.1		
			[1]			
12	(c)	outlier on extreme left oe should be removed as nobody could have a BMI this low oe	B1	2.4	need to refer to BMI being implausible	advantage
		outlier with (very) high pulse rate oe should not be removed as it is plausible oe	B1	2.4		advantage
			[2]			
12	(d)	no correlation	B1	2.2b	ignore other comments unless contradictory	
			[1]			
12	(e)	because the value of pmcc is close to 1 or because there is strong correlation oe	B1	2.2b		
			[1]			

Question		Answer	Marks	AOs	Guidance																						
12	(f)	86.784, 86.78 or 86.8	B1	3.4	isw																						
			[1]																								
13	(a)	mean = 1.5 BC	B1	1.1																							
		sd = 1.1, 1.10 or awrt 1.096 BC	B1	1.1																							
			[2]																								
13	(b)	$n = 10$ and $p = 0.15$	B1	3.3																							
			[1]																								
13	(c)	<table border="1"> <tr> <td>N</td> <td>0</td> <td>1</td> <td>2</td> <td>3</td> <td>4</td> <td>5 +</td> </tr> <tr> <td>O</td> <td>19</td> <td>35</td> <td>28</td> <td>13</td> <td>5</td> <td>0</td> </tr> <tr> <td>E</td> <td>19.7</td> <td>34.7</td> <td>27.6</td> <td>13.0</td> <td>4.0</td> <td>1.0</td> </tr> </table>	N	0	1	2	3	4	5 +	O	19	35	28	13	5	0	E	19.7	34.7	27.6	13.0	4.0	1.0	M1	3.4	use of B(10, p) FT their p to find at least 2 probabilities; may be unsimplified	must see at least 2 probabilities for M1
		N	0	1	2	3	4	5 +																			
		O	19	35	28	13	5	0																			
E	19.7	34.7	27.6	13.0	4.0	1.0																					
M1	3.4	multiplying their binomial probabilities by 100	must see at least 2 expected values calculated for M1																								
A1	1.1	all correct to 1 dp or better; must add up to 100																									
			[3]																								
13	(d)	close match between theoretical (or expected) and observed frequencies so model is a good fit	B1	3.5a	FT dependent on award of M1M1 in (c) and all frequencies calculated																						
			[1]																								
14	(a)	$2x(x - 2)^3 + x^2 \times 3(x - 2)^2$	M1	3.1a	product rule & chain rule; allow one error																						
			A1	1.1																							
		$(x - 2)$ identified as factor	M1	2.1																							
		$x(x - 2)^2(5x - 4)$ cao	A1	1.1																							
			[4]																								
		<i>Alternative</i> $5x^4 - 24x^3 + 36x^2 - 16x$	M1		NB from $x^5 - 6x^4 + 12x^3 - 8x^2$ expand brackets and differentiate; allow one error																						
			A1																								
		$(x - 2)$ identified as factor	M1																								
		$x(x - 2)^2(5x - 4)$ cao	A1																								
			[4]																								

Question		Answer	Marks	AOs	Guidance
14	(b)	their $\frac{dy}{dx} = 0$ so i	M1	2.1	
		$x = 0, 2, \frac{4}{5}$	A1	1.1	
		(0,0) and (2.0)	A1	1.1	
		$(0.8, -1.10592)$ or $(0.8, -\frac{3456}{3125})$	A1	2.2a	accept – 1.10592 to 2 sf or better
			[4]		
14	(c)	<p>2nd derivative = – 16 at (0,0) so max</p> <p>2nd derivative = 5.76 at (0.8, – 1.10592) so min</p> <p>eg gradient = 1 at $x = 1$ and 33 at $x = 3$ so inflection at $x = 2$</p> <p>eg $y = -1$ at $x = 1$ and $y = 9$ at $x = 3$ so inflection at $x = 2$</p> <p>NB 2nd derivative test is indecisive at $x = 2$</p>	<p>B1</p> <p>B1</p> <p>B1</p>	<p>1.1</p> <p>1.1</p> <p>3.1a</p>	<p>or for any of the three points: considers y or $\frac{dy}{dx}$ or $\frac{d^2y}{dx^2}$ either side of correct stationary point accompanied by suitable commentary; must see numerical values</p>
			[3]		
14	(d)		<p>M1</p> <p>A1</p>	<p>1.1</p> <p>1.1</p>	<p>shape of curve correct with max, min and inflection</p> <p>correct intercepts marked on sketch or identified next to graph</p>
			[2]		

Question		Answer	Marks	AOs	Guidance
15	(a)	$a = 0.01$ and $r = 0.99$	B1	2.1	
		a and r substituted in $\frac{a}{1-r}$ oe seen	M1	1.1	a or r must be correct
		$\frac{0.01}{1-0.99}$ oe = 1	A1	2.4	
			[3]		
15	(b)	$(n =) 312$	M1	3.1a	allow 313
		$\frac{0.01(1-0.99^n)}{1-0.99}$ or $\sum_1^n 0.99^{n-1} \times 0.01$ evaluated	M1	1.1	$n = 312$ or 313 ; condone $n = 6$
		awrt 0.9565 or $0.9570 > 0.95$ so model predicts Layla will beat him within 6 years oe	A1	3.4	
			[3]		
		<i>Alternative</i> $\frac{0.01(1-0.99^n)}{1-0.99} = 0.95$	M1		may use cdf from geometric distribution; allow $>$ or \geq instead of $=$
		$0.99^n = 0.05$	M1		attempt to simplify as far as “ $0.99^n =$ ”
		$n = 298.1$ weeks < 312 (or 313) so model predicts Layla will beat him within 6 years oe	A1		
			[3]		
15	(c)	Layla thinks she will improve (by practising), so she should become increasingly likely to beat Kofi oe or Layla thinks she is more likely to beat Kofi because he doesn't practise (but Layla does) oe	B1	3.5a	
			[1]		
15	(d)	probability of Layla winning increases as r increases oe	B1	2.4	B0 for eg probability of Layla winning increases exponentially
			[1]		

Question		Answer	Marks	AOs	Guidance	
15	(e)	$k(1 + 4 + 9 + 16 + 25 + 36 + 49 + 64) = 1$ soi	M1	3.3		
		$k = \frac{1}{204}$ (= 0.00490 ...)	A1	1.1		
		$P(X \leq 6) = \frac{91}{204}$ (0.44607 ...) < 0.95 so Layla's statement not consistent with her model oe	A1	3.5a		
			[3]			
16		$u = 1 + \sqrt{x}$	B1	3.1a	for use in substitution	No marks for attempts based solely on integration by parts
		$\frac{du}{dx} = \frac{1}{2}x^{-\frac{1}{2}}$	M1	1.1	allow M1 for $x^{-\frac{1}{2}}$	
		$\sqrt{x} = u - 1$	M1	1.1	allow sign error	
		$\int \frac{(u-1)^2 \times 2(u-1)}{u} du$	A1	3.1a	du may be seen later	
		$\frac{2(u^3 - 3u^2 + 3u - 1)}{u}$ soi	M1	2.1	allow sign errors and/or omission of 2	
		$2 \left[\frac{u^3}{3} - \frac{3u^2}{2} + 3u - \ln u \right]$	M1	3.1a	divides their cubic through by u and integrates dependent on award of first two M marks; allow sign errors and coefficient errors	must have $\ln u$
			A1	1.1		
		$\frac{2(1+\sqrt{x})^3}{3} - 3(1+\sqrt{x})^2 + 6(1+\sqrt{x}) - 2\ln(1+\sqrt{x})$ +c oe isw	A1	3.2a	if answer fully correct but either + c or du not seen then withhold final A1	
			[8]			

Question			Answer	Marks	AOs	Guidance	
			<i>Alternative</i> $x = u^2$	B1		<i>or</i> $u = \sqrt{x}$	may see $u = \frac{1}{1+\sqrt{x}}$ or $e^u = (1+\sqrt{x})$
			$\frac{dx}{du} = 2u$	M1		allow M1 for $\frac{du}{dx} = x^{-\frac{1}{2}}$	
			$\sqrt{x} = u$	M1		$x = u^2$	
			$\int \frac{u^2 \times 2u}{1+u} du$	A1			
			$(2u^2 - 2u + 2) - \frac{2}{1+u}$ from long division oe	M1		allow sign errors and/or omission of 2	
			$\frac{2u^3}{3} - u^2 + 2 - 2\ln(1+u)$	M1 A1		integration attempted; allow sign errors and coefficient errors	
			$\frac{2x\sqrt{x}}{3} - x + 2\sqrt{x} - 2\ln(1+\sqrt{x}) + c$	A1		if answer fully correct but either + c or du not seen then withhold final A1	
				[8]			

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